Recursive Function

Discrete Mathematics

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Recursively Defined Functions

Basis step: Specifies the value of the function for the first term(s).
Recursive step: Gives a rule for finding subsequent values using a previous value(s) beginning at those defined in the basis step.
Example:
If f is defined recursively by $f(0) = 2$ and $f(n+1) = 3f(n) - 1$.
Find $f(1)$, $f(2)$, $f(3)$, and $f(4)$.

$$f(1) = f(0+1) = 3f(0) - 1 = (3 \times 2) - 1 = 5$$
$$f(2) = f(1+1) = 3f(1) - 1 = (3 \times 5) - 1 = 14$$
$$f(3) = f(2+1) = 3f(2) - 1 = (3 \times 14) - 1 = 41$$
$$f(4) = f(3+1) = 3f(3) - 1 = (3 \times 41) - 1 = 122$$
$$f(5) = (3 \times 122) - 1 = 365$$

Recursively Defined Functions: Examples

Example 1:
Let g be defined recursively by $g(0) = 1$ and $g(n+1) = 2g(n) + 3$. Find $g(1)$, $g(2)$, $g(3)$, and $g(4)$.

$$g(1) = 2g(0) + 3 = 2 \times 1 + 3 = 5$$
$$g(2) = 2g(1) + 3 = 2 \times 5 + 3 = 13$$
$$g(3) = 2g(2) + 3 = 2 \times 13 + 3 = 29$$
$$g(4) = 2g(3) + 3 = 2 \times 29 + 3 = 61$$

Recursively Defined Functions: Examples

Example 2:
Let h be defined recursively by $h(0) = 2$ and $h(n+1) = 3h(n) - 1$. Find $h(1)$, $h(2)$, $h(3)$, and $h(4)$.

$$h(1) = 3h(0) - 1 = 3 \times 2 - 1 = 5$$
$$h(2) = 3h(1) - 1 = 3 \times 5 - 1 = 14$$
$$h(3) = 3h(2) - 1 = 3 \times 14 - 1 = 41$$
$$h(4) = 3h(3) - 1 = 3 \times 41 - 1 = 122$$

Recursively Defined Functions: Examples

Example 3:
Let k be defined recursively by $k(0) = 1$ and $k(n+1) = k(n) + 2^n$. Find $k(1)$, $k(2)$, $k(3)$, and $k(4)$.

$$k(1) = k(0) + 2^0 = 1 + 1 = 2$$
$$k(2) = k(1) + 2^1 = 2 + 2 = 4$$
$$k(3) = k(2) + 2^2 = 4 + 4 = 8$$
$$k(4) = k(3) + 2^3 = 8 + 8 = 16$$