Lecture Note on Modified Secant Method

Numerical Methods

Dr. Mohamad Salah Uddin

Learning Objectives

- Understand the idea of the Modified Secant Method.
- Apply the method using one initial guess.
- Use a constant perturbation factor δ.
- Calculate approximate relative error after each iteration.
- Solve nonlinear equations step by step.

1 Introduction

The Modified Secant Method is a numerical method for finding a root of a nonlinear equation. It is a variation of the Secant Method. The ordinary Secant Method needs two initial guesses. The Modified Secant Method needs only one initial guess.

Let the nonlinear equation be

$$f(x) = 0.$$

The method estimates the slope near the current approximation by using a small perturbation. This perturbation is controlled by a constant value δ.

2 Formula of the Modified Secant Method

The formula is

$$x_{i+1} = x_i - \frac{\delta x_i f(x_i)}{f(x_i + \delta x_i) - f(x_i)}.$$

Here,

- x_i is the current approximation,
- x_{i+1} is the next approximation,
- δ is the constant perturbation factor,
- $f(x_i)$ is the function value at x_i,
- $f(x_i + \delta x_i)$ is the function value at the perturbed point.

3 Constant Perturbation Factor

In this lecture, δ is always treated as a constant. We use

$$\boxed{\delta = 0.01}.$$

This value remains fixed for every iteration. It does not change during the calculation. The perturbed point is

$$x_i + \delta x_i.$$

Since $\delta = 0.01$, we get

$$x_i + \delta x_i = x_i + 0.01x_i.$$

$$x_i + \delta x_i = x_i(1 + 0.01).$$

Therefore,

$$\boxed{x_i + \delta x_i = 1.01x_i}.$$

Important Note: Only x_i changes from iteration to iteration. The value of δ remains fixed as 0.01 throughout the whole problem.

4 Approximate Relative Error

The approximate relative error is calculated by

$$\varepsilon_a = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100\%.$$

This error helps us decide whether the approximation is good enough.

5 Stopping Criterion

The iteration can be stopped when

$$\varepsilon_a < \varepsilon_s,$$

where ε_s is the specified stopping error.
For example, if

$$\varepsilon_s = 0.5\%,$$

then the method stops when

$$\varepsilon_a < 0.5\%.$$

6 Solved Example 1

Use the Modified Secant Method to find a root of

$$f(x) = x^3 - 3x^2 + 3.119 \times 10^{-4}$$

using

$$x_0 = 0.05, \qquad \delta = 0.01.$$

Here, $\delta = 0.01$ is constant for all iterations.

Iteration 1

$$x_0 = 0.05$$

$$f(0.05) = (0.05)^3 - 3(0.05)^2 + 3.119 \times 10^{-4}$$

$$f(0.05) = 0.000125 - 3(0.0025) + 0.0003119$$

$$f(0.05) = 0.000125 - 0.0075 + 0.0003119$$

$$f(0.05) = -0.0070631$$

Perturbed point:

$$x_0 + \delta x_0 = 0.05 + 0.01(0.05)$$

$$x_0 + \delta x_0 = 0.0505$$

$$f(0.0505) = (0.0505)^3 - 3(0.0505)^2 + 0.0003119$$

$$f(0.0505) = -0.007210062$$

Now apply the formula:

$$x_1 = 0.05 - \frac{0.01(0.05)(-0.0070631)}{-0.007210062 - (-0.0070631)}$$

$$x_1 = 0.05 - \frac{-0.00000353155}{-0.000146962}$$

$$x_1 = 0.05 - 0.0240303$$

$$x_1 = 0.0259697$$

Approximate relative error:

$$\varepsilon_a = \left| \frac{0.0259697 - 0.05}{0.0259697} \right| \times 100$$

$$\varepsilon_a = 92.53\%$$

Iteration 2

$$x_1 = 0.0259697$$

$$f(0.0259697) = (0.0259697)^3 - 3(0.0259697)^2 + 0.0003119$$

$$f(0.0259697) = -0.001693861$$

Perturbed point:

$$x_1 + \delta x_1 = 0.0259697 + 0.01(0.0259697)$$

$$x_1 + \delta x_1 = 1.01(0.0259697)$$

$$x_1 + \delta x_1 = 0.0262294$$

$$f(0.0262294) = -0.001733998$$

Now,

$$x_2 = 0.0259697 - \frac{0.01(0.0259697)(-0.001693861)}{-0.001733998 - (-0.001693861)}$$

$$x_2 = 0.0259697 - \frac{-0.0000004399}{-0.0000401371}$$

$$x_2 = 0.0259697 - 0.0109597$$

$$x_2 = 0.0150100$$

Approximate relative error:

$$\varepsilon_a = \left| \frac{0.0150100 - 0.0259697}{0.0150100} \right| \times 100$$

$$\varepsilon_a = 73.02\%$$

Iteration 3

$$x_2 = 0.0150100$$

$$f(0.0150100) = -0.000360619$$

Perturbed point:

$$x_2 + \delta x_2 = 1.01(0.0150100)$$

$$x_2 + \delta x_2 = 0.0151601$$

$$f(0.0151601) = -0.000374102$$

Now,

$$x_3 = 0.0150100 - \frac{0.01(0.0150100)(-0.000360619)}{-0.000374102 - (-0.000360619)}$$

$$x_3 = 0.0150100 - \frac{-0.00000005413}{-0.000013483}$$

$$x_3 = 0.0150100 - 0.0040146$$

$$x_3 = 0.0109954$$

Approximate relative error:

$$\varepsilon_a = \left|\frac{0.0109954 - 0.0150100}{0.0109954}\right| \times 100$$

$$\varepsilon_a = 36.51\%$$

Iteration 4

$$x_3 = 0.0109954$$

$$f(0.0109954) = -0.0000494697$$

Perturbed point:

$$x_3 + \delta x_3 = 1.01(0.0109954)$$

$$x_3 + \delta x_3 = 0.0111054$$

$$f(0.0111054) = -0.0000567197$$

Now,

$$x_4 = 0.0109954 - \frac{0.01(0.0109954)(-0.0000494697)}{-0.0000567197 - (-0.0000494697)}$$

$$x_4 = 0.0109954 - \frac{-0.000000005439}{-0.000007250}$$

$$x_4 = 0.0109954 - 0.0007503$$

$$x_4 = 0.0102452$$

Approximate relative error:

$$\varepsilon_a = \left|\frac{0.0102452 - 0.0109954}{0.0102452}\right| \times 100$$

$$\varepsilon_a = 7.32\%$$

Iteration 5

$$x_4 = 0.0102452$$

$$f(0.0102452) = -0.000001915$$

Perturbed point:

$$x_4 + \delta x_4 = 1.01(0.0102452)$$

$$x_4 + \delta x_4 = 0.0103476$$

$$f(0.0103476) = -0.000008212$$

Now,

$$x_5 = 0.0102452 - \frac{0.01(0.0102452)(-0.000001915)}{-0.000008212 - (-0.000001915)}$$

$$x_5 = 0.0102452 - \frac{-0.0000000001962}{-0.0000062967}$$

$$x_5 = 0.0102452 - 0.00003116$$

$$x_5 = 0.0102140$$

Approximate relative error:

$$\varepsilon_a = \left| \frac{0.0102140 - 0.0102452}{0.0102140} \right| \times 100$$

$$\varepsilon_a = 0.305\%$$

Result Table for Example 1

Iteration	x_i	$f(x_i)$	Approx. Relative Error
0	0.050000	-0.0070631	–
1	0.0259697	-0.0016939	92.53%
2	0.0150100	-0.0003606	73.02%
3	0.0109954	-0.0000495	36.51%
4	0.0102452	-0.0000019	7.32%
5	0.0102140	≈ 0	0.305%

Table 1: Modified Secant Method results for Example 1

Therefore, the approximate root is

$$\boxed{x \approx 0.010214}.$$

7 Solved Example 2

Use the Modified Secant Method to find a root of

$$f(x) = x^2 - 4$$

using

$$x_0 = 3, \qquad \delta = 0.01.$$

Perform iterations until the approximate relative error is less than 0.5%.
Here also, $\delta = 0.01$ is constant for all iterations.

Iteration 1

$$x_0 = 3$$

$$f(3) = 3^2 - 4$$

$$f(3) = 9 - 4 = 5$$

Perturbed point:

$$x_0 + \delta x_0 = 3 + 0.01(3)$$

$$x_0 + \delta x_0 = 3.03$$

$$f(3.03) = (3.03)^2 - 4$$

$$f(3.03) = 9.1809 - 4$$

$$f(3.03) = 5.1809$$

Now apply the formula:

$$x_1 = 3 - \frac{0.01(3)(5)}{5.1809 - 5}$$

$$x_1 = 3 - \frac{0.15}{0.1809}$$

$$x_1 = 3 - 0.8292$$

$$x_1 = 2.1708$$

Approximate relative error:

$$\varepsilon_a = \left| \frac{2.1708 - 3}{2.1708} \right| \times 100$$

$$\varepsilon_a = 38.20\%$$

Iteration 2

$$x_1 = 2.1708$$

$$f(2.1708) = (2.1708)^2 - 4$$

$$f(2.1708) = 0.7124$$

Perturbed point:

$$x_1 + \delta x_1 = 1.01(2.1708)$$

$$x_1 + \delta x_1 = 2.1925$$

$$f(2.1925) = (2.1925)^2 - 4$$

$$f(2.1925) = 0.8069$$

Now,

$$x_2 = 2.1708 - \frac{0.01(2.1708)(0.7124)}{0.8069 - 0.7124}$$

$$x_2 = 2.1708 - \frac{0.01546}{0.0945}$$

$$x_2 = 2.1708 - 0.1636$$

$$x_2 = 2.0072$$

Approximate relative error:

$$\varepsilon_a = \left| \frac{2.0072 - 2.1708}{2.0072} \right| \times 100$$

$$\varepsilon_a = 8.15\%$$

Iteration 3

$$x_2 = 2.0072$$

$$f(2.0072) = (2.0072)^2 - 4$$

$$f(2.0072) = 0.0289$$

Perturbed point:

$$x_2 + \delta x_2 = 1.01(2.0072)$$

$$x_2 + \delta x_2 = 2.0273$$

$$f(2.0273) = (2.0273)^2 - 4$$

$$f(2.0273) = 0.1099$$

Now,

$$x_3 = 2.0072 - \frac{0.01(2.0072)(0.0289)}{0.1099 - 0.0289}$$

$$x_3 = 2.0072 - \frac{0.000580}{0.0810}$$

$$x_3 = 2.0072 - 0.00716$$

$$x_3 = 2.0000$$

Approximate relative error:

$$\varepsilon_a = \left| \frac{2.0000 - 2.0072}{2.0000} \right| \times 100$$

$$\varepsilon_a = 0.36\%$$

Since

$$0.36\% < 0.5\%,$$

we stop the iteration.

Result Table for Example 2

Iteration	x_i	$f(x_i)$	Approx. Relative Error
0	3.0000	5.0000	–
1	2.1708	0.7124	38.20%
2	2.0072	0.0289	8.15%
3	2.0000	≈ 0	0.36%

Table 2: Modified Secant Method results for Example 2

Therefore, the approximate root is

$$\boxed{x \approx 2}.$$

8 Algorithm for Students

Step 1: Choose the function $f(x)$.

Step 2: Choose one initial guess x_0.

Step 3: Choose the constant perturbation factor δ. In this lecture, $\delta = 0.01$.

Step 4: Calculate $f(x_i)$.

Step 5: Calculate the perturbed point:

$$x_i + \delta x_i = 1.01x_i.$$

Step 6: Calculate $f(x_i + \delta x_i)$.

Step 7: Apply the Modified Secant formula:

$$x_{i+1} = x_i - \frac{\delta x_i f(x_i)}{f(x_i + \delta x_i) - f(x_i)}.$$

Step 8: Calculate approximate relative error:

$$\varepsilon_a = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100\%.$$

Step 9: Repeat until the error is less than the stopping error.

9 Class Practice Problems

Solve the following using the Modified Secant Method. Use $\delta = 0.01$ as a constant in all problems.

1. Solve
$$f(x) = x^3 - x - 2$$
using $x_0 = 1.5$. Stop when $\varepsilon_a < 0.5\%$.

2. Solve
$$f(x) = e^{-x} - x$$
using $x_0 = 0.5$. Stop when $\varepsilon_a < 0.5\%$.

3. Solve
$$f(x) = \cos x - x$$
using $x_0 = 0.7$. Stop when $\varepsilon_a < 0.5\%$.

4. Solve
$$f(x) = x^3 - 4x - 9$$
using $x_0 = 2.5$. Stop when $\varepsilon_a < 0.5\%$.

10 Conclusion

The Modified Secant Method is useful because it needs only one initial guess. It uses a constant perturbation factor to estimate the slope near the current point. In this lecture, we used $\delta = 0.01$ for all iterations. The value of δ remained fixed. Only the approximation x_i changed from one iteration to the next.